$$(AB)C = A(BC)$$

$$\underbrace{\underbrace{\underset{m\times n}{A}\ \underset{n\times p}{B}}_{m\times p}\ \underset{p\times q}{C}}_{m\times q} \qquad \underbrace{\underset{m\times n}{A}\ \underbrace{\underset{n\times p}{B}\ \underset{p\times q}{C}}_{n\times q}}_{m\times q}$$

IF # COLUMNS IN A = # ROWS IN B,

ENTRY IN ROW i COLUMN j OF AB

IS

THE DOT PRODUCT OF ROW i OF A WITH COLUMN j OF B

$$A(B+C) = AB + AC$$
$$(B+C)A = BA + CA$$

$$A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \quad C = \begin{bmatrix} -5 & 4 \\ 0 & 2 \end{bmatrix}$$

$$AB \neq BA$$

$$\underset{2\times2\ 2\times2}{BA} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2+-2 & 6+-6 \\ -4+4 & -12+12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O_{2\times2}$$

NOTE. $BA = O$ BUT $B \neq O$, $A \neq O$ (NO ZERO PRODUCT PROPERTY)

$$AB = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -10 & 20 \\ -5 & 10 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -10 & 20 \\ -5 & 10 \end{bmatrix}$$

NOTE:

$AB = AC$

BUT

$A \neq O$ AND $B \neq C$

(NO CANCELLATION)

A MATRIX IS SQUARE IF IT HAS THE SAME NUMBER OF ROWS AND COLUMNS

IE. ORDER IS $n \times n$

$$\underset{2\times 2}{\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}} \qquad \underset{3\times 3}{\begin{bmatrix} 3 & 4 & 1 \\ -2 & 4 & 3 \\ -1 & -3 & 2 \end{bmatrix}} = B$$

AN IDENTITY MATRIX I_3 IS A SQUARE MATRIX WITH 1's ALONG THE MAIN DIAGONAL AND 0's EVERYWHERE ELSE

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underset{m\times m}{\overset{I_m}{I}}\ \underset{m\times n}{A} = \underset{m\times n}{A}\ \underset{n\times n}{\overset{I_n}{I}} = \underset{m\times n}{A}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underset{3\times 3}{I_3}\ \underset{3\times 3}{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 4 & 1 \\ -2 & 4 & 3 \\ -1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 1 \\ -2 & 4 & 3 \\ -1 & -3 & 2 \end{bmatrix} = B$$

ORDER OF I_3B

SOLVE $\underset{2\times 2}{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} \underset{2\times 1}{X} = \underset{2\times 1}{\begin{bmatrix} 5 \\ 3 \end{bmatrix}}$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} X_1 + X_2 \\ X_1 - X_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$X_1 + X_2 = 5$$
$$X_1 - X_2 = 3$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & -1 & 3 \end{array}\right] R_2 + (-1)R_1$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -2 & -2 \end{array}\right] R_2 * (-\tfrac{1}{2})$$

REF $\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 1 \end{array}\right] R_1 + (-1)R_2$

RREF $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \end{array}\right]$ NOTE! THIS IS I

CHECK:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \checkmark$$

$$X = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

8.4 DETERMINANT

(OF A SQUARE MATRIX)

$2(2x+y=(4)2$

$4x-2y=7$

$4x+2y=8$

$8x=15$

$x=\frac{15}{8}$

$2\cdot\frac{15}{8}+y=4$

$y=\frac{1}{4}$

$(x,y)=(\frac{15}{8},\frac{1}{4})$

ONE SOLUTION

SYSTEM IS CONSISTENT

$-2(2x+y)=(4)(-2)$

$4x+2y=8$

$-4x-2y=-8$

$0=0$

so $2x+y=4$

$y=4-2x$ IS A SOL'N FOR ALL X

$(x,y)=(x,4-2x)$

INFINITELY MANY SOL'NS

SYSTEM IS DEPENDENT

$-2(2x+y)=(4)(-2)$

$4x+2y=7$

$-4x-2y=-8$

$0=-1$

SO NO SOL'NS

SYSTEM IS INCONSISTENT

IF $\begin{matrix} ax+by=e \\ cx+dy=f \end{matrix}$ ARE MULTIPLES $(ax+by=K(cx+dy))$

$\frac{a}{c}=\frac{b}{d}$

$ad=bc$

$ad-bc=0$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} e \\ f \end{bmatrix}$$

DETERMINANT OF $\begin{bmatrix} a & b \\ c & d \end{bmatrix}=ad-bc$

IF DETERMINANT=0 SYSTEM IS EITHER INCONSISTENT OR DEPENDENT

$$2x + y - 3z = 1$$
$$x - 2y + z = 2$$
$$3x + 3y - z = 6$$

$$\underbrace{\begin{bmatrix} 2 & 1 & -3 \\ 1 & -2 & 1 \\ 3 & 3 & -1 \end{bmatrix}}_{\text{DETERMINANT}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 1 & -3 \\ 1 & -2 & 1 \\ 3 & 3 & -1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 1 & -2 \\ 3 & 3 \end{matrix} = 4+3-9+1-6-18$$
$$= 7-8-24$$
$$= -1-24$$
$$= -25 \neq 0$$

SYSTEM IS CONSISTENT